Application of multi-objective differential evolution algorithm (MDEA) to irrigation planning

Josiah Adeyemo*, Fred Otieno

Josiah Adeyemo

Tshwane University of Technology
We empower people

INTRODUCTION

- Farmers in a water scarce environment have a problem of maximizing total income from farming.
- Proper irrigation planning can help achieve this aim
- Irrigation planning is a multi-objective optimization problem
- Irrigation can be planned using mathematical optimization techniques
- Differential evolution algorithm can be used.

Differential Evolution (DE)

- An improved version of Genetic Algorithm (GA),
- A type of Evolutionary Algorithm (EA) for faster optimization
- Difference between GA and DE; GA rely on crossover, - a mechanism of probabilistic and useful exchange of information among solutions to locate better solutions
- DE uses a non uniform crossover.

Differential Evolution (DE)

- Real coding of floating point numbers.
- Simple structure, ease of use, speed and robustness.
- Generating trial parameter vectors.
- P1+(P2-P3)
- Key parameters of control are:
  - NP – the population size,
  - CR – the crossover constant and
  - F – the weight applied to random differential scaling factor.

DE population structure

- DE population structure is such that contain NP, D-dimensional vectors of real-valued parameters
- $P_{x,g} = (x_{i,g})$, i = 0, 1 …NP – 1, g = 0, 1, …gmax
- $x_{i,g} = (x_{i,g})$, j = 0, 1 …D – 1

where

- NP – Number of population
- g_{max} – Number of generation
- D – Number of parameters

DE Algorithm

- Initialization

$$X_{j,i,0} = rand_j(0,1) \,.\, (b_{j,u} - b_{j,L}) + b_{j,L}$$

- Where $rand_j(0,1)$= uniformly distributed random number within [0,1].

DE Mutation Scheme

- 3 vectors are randomly chosen i.e. X_{r1}, X_{r2} &X_{r3}.

$$V_{i,g} = X_{r0,g} + F.\,(X_{r1,g} - X_{r2,g})$$

- $F \in (0,1^{+})$

F controls the rate at which the population evolves.

DE Crossover Scheme

$$U_{i,g} = U_{j,i,g} = \begin{cases} v_{j,i,g} \text{ if } (rand_j(0,1) \leq Cr \text{ or } j = jrand \\ x_{j,i,g} \text{ otherwise} \end{cases}$$

- $Cr \in (0,1)$ =crossover probability.
- $U_{i,g}$ = trial vector,
- $x_{i,g}$, = target vector,
- By this, DE more tightly integrate recombination &selection than other EAs.

In summary, DE's generate-and-test loop is

$$u_{j,i,g} = \begin{cases} x_{i,r0,g} + F.(x_{i,r1,g} - x_{i,r2,g}), \text{if } (rand_j(0,1) \le Cr \text{ or } j = j_{rand}), \\ x_{j,i,g} \text{ otherwise} \end{cases}$$

$j = 0, 1, \ldots, D-1$; $j_{rand} \in \{0,1,\ldots,D-1\}$

$i = 0,1,\ldots,Np-1$

$g = 0,1,\ldots g_{max}$

$r0, r1, r2 \in \{0,1,\ldots,Np-1\}$, $r0 \ne r1 \ne r2 \ne i$

$$x_{i,g+1} = \begin{cases} u_{i,g} \text{ if } f(u_{i,g}) \le f(x_{i,g}) \\ x_{j,i,g} \text{ otherwise} \end{cases}$$

The 10 Strategies of DE

DE/x/y/z

DE	=	Differential Evolution,
x	=	a string which denotes the vector to be perturbed,
y	=	the no of different vectors for perturbation of x
z	=	the crossover method.

The 10 Strategies of DE

- DE/rand/1/bin
- DE/best/1/bin
- DE/best/2/bin
- DE/ran/2/bin
- DE/randtobest/1/bin

- DE/rand/1/exp
- DE/best/1/exp
- DE/best/2/exp
- DE/rand/2/exp
- DE/rand to best/1/exp.

Multi-objective differential evolution algorithm (MDEA)

- It follows the proposed DE algorithm but different in the implementation of multi-objectives.
- MDEA can be used on any strategy of DE but DE/rand/1/bin and DE/rand/1/exp are used in this study

Description of MDEA

- The vectors are randomly generated to create initial vectors and solutions to the problem.
- The generated solutions are allowed to undergo mutation, crossover and selection for the specified number of generations.
- The solutions that evolve are checked for domination and the dominated solutions are removed.
- The trial solution survives to the next generation if its objective function is better or equal in all the objectives.
- To handle constraints, if any of the constraints is violated, a high value is added to the objective function value to make the solution infeasible.

IRRIGATION PLANNING MODEL

Objective Function 1: Minimize the total irrigation water

- The total irrigation water released through the main canal to the farmers is minimized. This can be expressed as*:*

Min

$$TIR = \sum_{i=1}^{N} IR_i \qquad (1)$$

$$IR_i = \sum_{j=1}^{M} \left(\left(CWR_{i,j} * A_j \right) / 10 \right) \qquad (2)$$

Where,

- TIR = total irrigation release for the 12 months
- N = number of months (12) (January to December)
- IRi = irrigation release for month i
- M = number of cultivated lands (10)
- CWRi,j= crop water requirement on cultivated land j in month, i (mm)
- Aj = cultivated land j (ha)

IRRIGATION PLANNING MODEL

Objective Function 2: Maximize the total cultivated land

Maximize $$TA = \sum_{j=1}^{M} A_j$$

The cultivated land area available for irrigation is maximized to increase employment generation in the area.

Where TA is the total cultivated area in hectares

IRRIGATION PLANNING MODEL

Objective Function 3: Maximize the total benefit

Maximize

$$TB = \left(\sum_{i=1}^{M} \sum_{j=1}^{P} \left(TI_{i,j} * A_i \right) \right) - cw * TIR$$

$$TI_{i,j} = \left(\Pr ice_{i,j} * Yield_{i,j} \right) - Exp_{i,j}$$

Where,

- M = number of cultivated lands
- P = number of crops on each cultivated land
- TIi,j = total income of crop j on land i
- Am = cultivated land (ha)
- Cw = cost of irrigation water which is 8.77 cents per m3
- Pricei,j = selling price of crop j on land i (ZAR/ton)
- Yieldi,j = Yield of crop j on land m (tons/ha)
- Expi,j = expenses of crop j on land i (ZAR/ha)

IRRIGATION PLANNING MODEL

Constraint 1: Canal Capacity

- The monthly irrigation release should be less than the canal capacity.
- $V_i \leq$ canal capacity $\forall\ i = 1$ to 12

Constraint 2: Crop Water Requirement

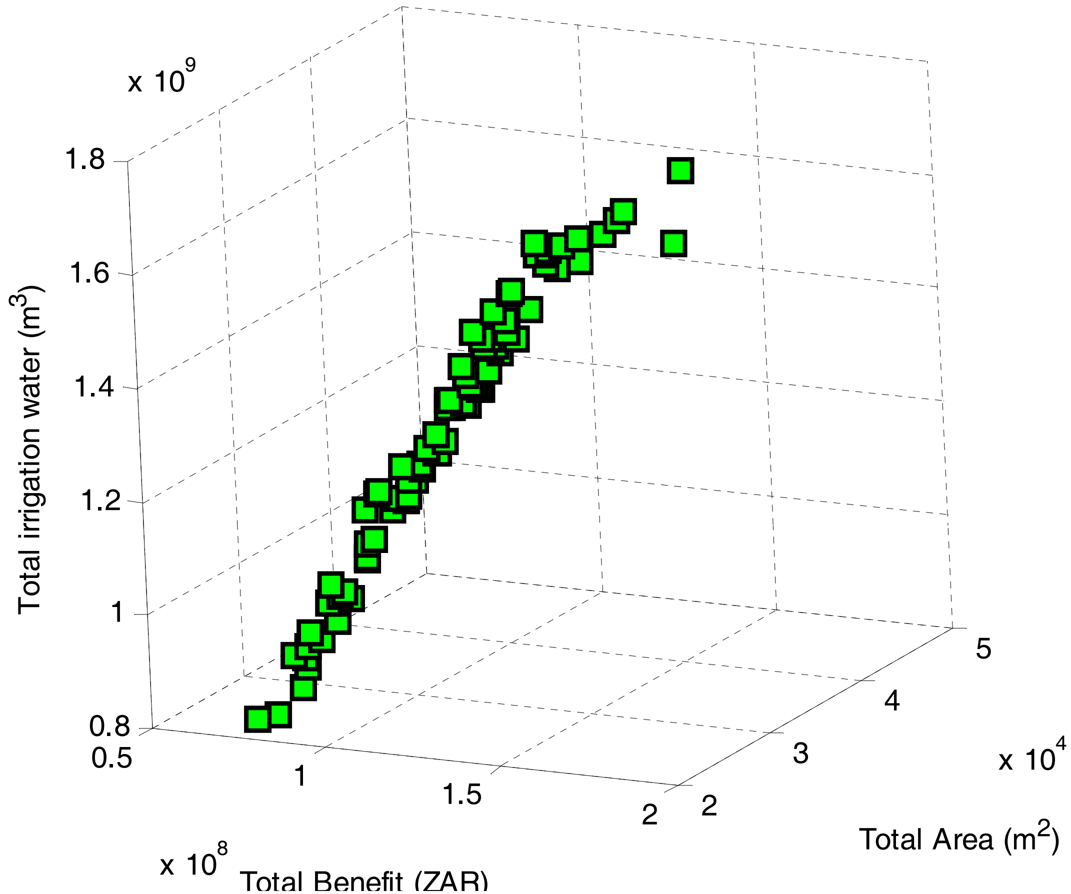
- Monthly irrigation release must meet the crop water requirements for all the crops in the month.
- $V_i \geq (CWR_{i,j} * A_j)$ $\forall\ i = 1$ to 12

Constraint 3: Minimum and Maximum Cultivated Areas

$5000 \leq A_j \leq A_{jmax}$

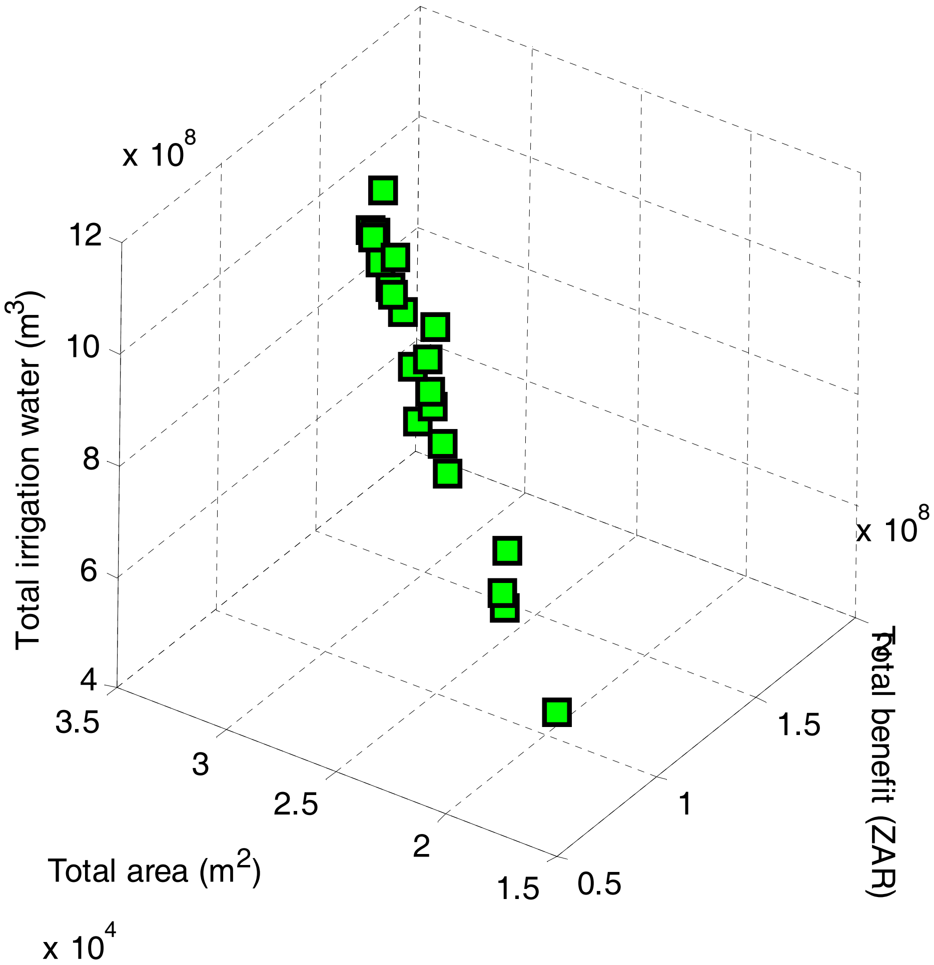
- where, A_{jmax} is the maximum area where each crop should be grown.

IRRIGATION PLANNING MODEL



•Figure 1: Pareto optimal set using MDEA1

IRRIGATION PLANNING MODEL



•Figure 1: Pareto optimal set using MDEA3

IRRIGATION PLANNING MODEL

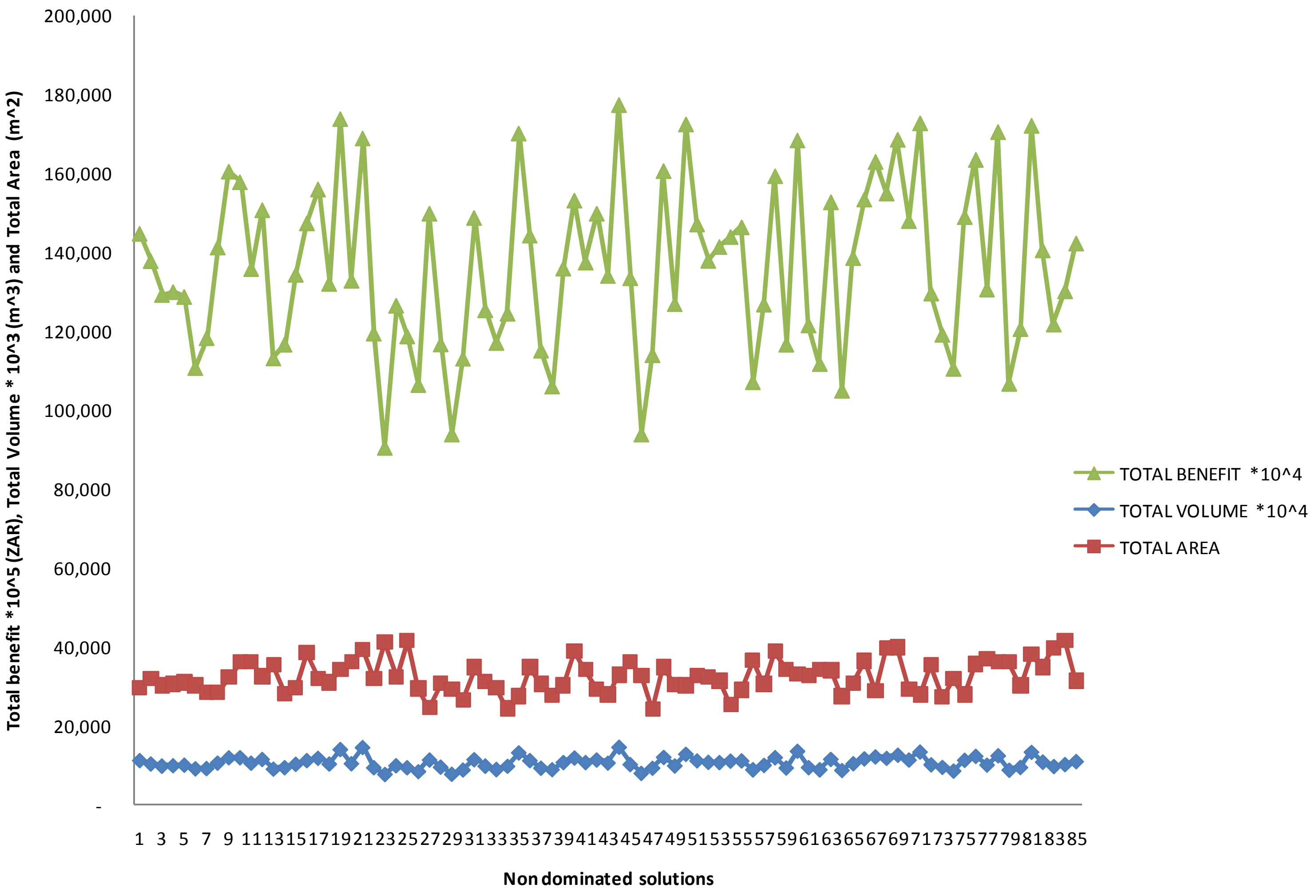


Figure 3: Total benefit, total area and total volume for the non-dominated solutions using MDEA1

IRRIGATION PLANNING MODEL

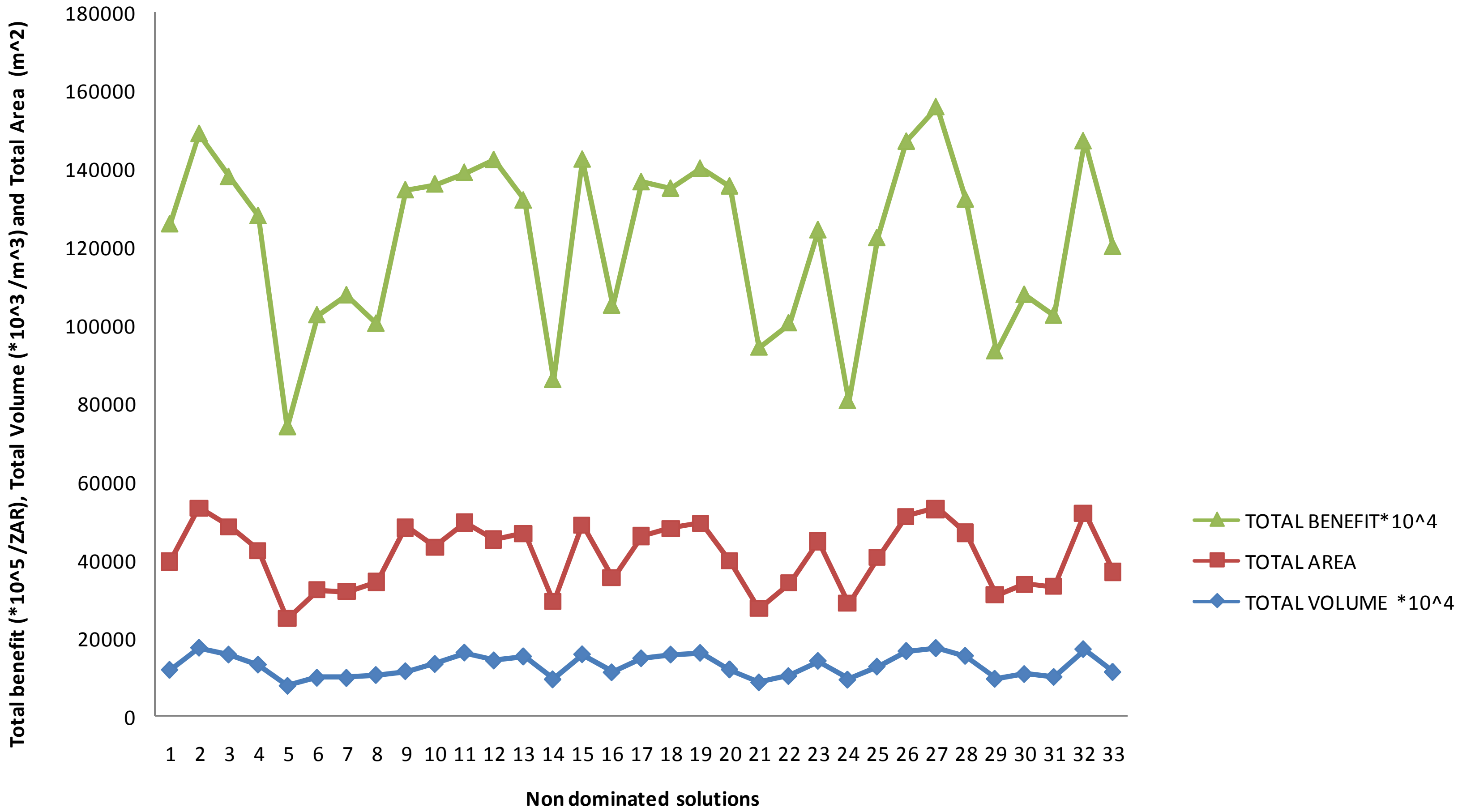


Figure 4: Total benefit, total area and total volume for the non-dominated solutions using MDEA3

oria

Results and Discussions

❖MDEA is used to solve the multi-objective problems above.

❖Results presented in Figures 1, 2, 3 and 4 show the non dominated solutions that converge to Pareto front.

❖2 goals in multi-objective optimization problem are achieved

To discover solutions as close to the Pareto-front as possible

To find solutions as diverse as possible in the obtained non-dominated front

❖All the problems converge to Pareto optimal front in less than 200 iterations

Conclusions

❖MDEA has been shown to be capable of solving multi-objective mathematical problems as well as an irrigation planning model

❖Dominated solutions are removed in the last generation only instead of removing them in all the generation.

❖MDEA saves a lot of simulation runs and also increases the number of non-dominated solutions giving wider Pareto optimal fronts.

Conclusions

❖MDEA gives good spread of solutions maintaining diversity and convergence

❖MDEA is a good alternative for solving water resources management problems and other engineering problems.

Questions?